# Distribution Tool Box

| **Discrete distributions** | **Random variable (z)** | **Parameters** | **Moments** | **R functions** | **JAGS functions** | Conjugate relationship |
|---|---|---|---|---|---|---|
| Poisson<br>$[z\|\lambda] =$<br>$\frac{\lambda^z e^{-\lambda}}{z!}$ | Counts of things that occur randomly over time or space, e.g., the number of birds in a forest stand, the number of fish in a kilometer of river, the number of prey captured per minute. | $\lambda$, the mean number of occurrences per time or space<br>$\lambda = \mu$ | $\mu = \lambda$<br>$\sigma^2 = \lambda$ | `dpois(x, lambda, log = FALSE)`<br>`ppois(q, lambda)`<br>`qpois(p, lambda),`<br>`rpois(n, lambda)` | `y[i] ~ dpois(lambda)` | $P(\lambda\|\mathbf{y}) =$<br>$\text{gamma}\left(\alpha + \sum_{i=1}^{n} y_i, \beta + n\right)$ |
| Binomial<br>$[z \mid \eta, \phi] =$<br>$\binom{\eta}{z}\phi^z\left(1-\phi\right)^{\eta-z}$<br>$\binom{\eta}{z} = \frac{\eta!}{z!(\eta-z)!}$<br>$[z \mid \eta, \phi] \propto$<br>$\phi^z\left(1-\phi\right)^{\eta-z}$ | Number of "successes" on a given number of trials, e.g., number of survivors in a sample of individuals, number of plots containing an exotic species from a sample, number of terrestrial pixels that are vegetated in an image. | $\eta$, the number of trials<br>$\phi$, the probability of a success<br>$\phi = 1 - \sigma^2/\mu$<br>$\eta = \mu^2/(\mu - \sigma^2)$ | $\mu = \eta\phi$<br>$\sigma^2 = \eta\phi\left(1-\phi\right)$ | `dbinom(x, size, prob, log = FALSE)`<br>`pbinom(q, size, prob)`<br>`qbinom(p, size, prob)`<br>`rbinom(n, size, prob)` | `y[i] ~ dbin(p,n)` | $P(p\|\mathbf{y}) =$<br>$\text{beta}\left(\alpha + y, \beta + n - y\right)$ |
| Bernoulli<br>$[z\|\phi] = \phi^z\left(1-\phi\right)^{1-z}$ | A special case of the binomial where the number of trials = 1 and the random variable can take on values 0 or 1. Widely used in survival analysis, occupancy models. | $\phi$, the probability that the random variable = 1<br>$\phi = \mu$<br>$\phi = 1/2 + 1/2\sqrt{1-4\sigma^2}$ | $\mu = \phi$<br>$\sigma^2 = \phi\left(1-\phi\right)$ | `dbinom(x, size=1, prob, log = FALSE)`<br>`pbinom(q, size=1, prob)`<br>`qbinom(p, size=1, prob)`<br>`rbinom(n, size=1, prob)`<br>Note that size *must* = 1. | `y[i]~dbern(p)` | |
| Multinomial<br>$[\mathbf{z} \mid \eta, \phi] =$<br>$\eta! \prod_{i=1}^{k} \frac{\phi_i^{z_i}}{z_i}$ | Counts that fall into $k > 2$ categories, e.g., number of individuals in age classes, number of pixels in different landscape categories, number of species in trophic categories in a sample from a food web. | $\mathbf{z}$ a vector giving the number of counts in each category, $\phi$ a vector of the probabilities of occurrence in each category,<br>$\sum_{i=1}^{k}\phi_i = 1$,<br>$\sum_{i=1}^{k} z_i = \eta$ | $\mu_i = \eta\phi_i$<br>$\sigma_i^2 =$<br>$\eta\phi_i\left(1-\phi_i\right)$ | `rmultinom(n, size, prob)`<br>`dmultinom(x, size, prob, log = FALSE)` | `y[i,]~dmulti(p[],n)` | |

| **Continuous Distributions** | **Random variable (z)** | **Parameters** | **Moments** | **R functions** | **JAGS function** | **Conjugate prior for** | **Vague Prior** |
|---|---|---|---|---|---|---|---|
| Normal<br>$[z\|\mu,\sigma^2] =$<br>$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(z-\mu)^2}{2\sigma^2}}$ | Continuously distributed quantities that can take on positive or negative values. Sums of things. | $\mu, \sigma^2$ | $\mu, \sigma^2$ | `dnorm(x, mean, sd, log = FALSE)`<br>`pnorm(q, mean, sd)`<br>`qnorm(p, mean, sd)`<br>`rnorm(n, mean, sd)` | `# tau = 1/sigma^2#`<br>`#likelihood`<br>`y[i]~dnorm(mu,tau)`<br>`#prior`<br>`theta ~ dnorm(mu,tau)` | normal mean (with known variance) | `dnorm(0,1E-6)`<br>#This is scale dependent. The larger the parameter value, the smaller tau must be to make the prior uninformative. |
| Lognormal<br>$[z \mid \alpha, \beta]$<br>$\frac{1}{z\sqrt{2\pi\beta^2}} e^{-\frac{(\ln z-\alpha)^2}{2\beta^2}}$ | Continuously distributed quantities with non-negative values. Random variables that have the property that their logs are normally distributed. Thus if $z$ is normally distributed then $\exp(z)$ is lognormally distributed. Represents products of things. The variance increases with the mean squared. | $\alpha$, the mean of $z$ on the log scale<br>$\beta$, the standard deviation of $z$ on the log scale<br>$\alpha = \log(\text{median}(z))$<br>$\alpha = \ln(\mu) - 1/2 \ln\left(\frac{\sigma^2+\mu^2}{\mu^2}\right)$<br>$\beta = \sqrt{\ln\left(\frac{\sigma^2+\mu^2}{\mu^2}\right)}$ | $\mu = e^{\alpha+\frac{\beta^2}{2}}$<br>$\text{median}(z) = e^{\alpha}$<br>$\sigma^2 = \left(e^{\beta^2}-1\right) e^{2\alpha+\beta^2}$ | `dlnorm(x, meanlog, sdlog)`<br>`plnorm(q, meanlog, sdlog)`<br>`qlnorm(p, meanlog, sdlog)`<br>`rlnorm(n, meanlog, sdlog)` | `#likelihood`<br>`y[i]~dlnorm(alpha,tau)`<br>`#prior`<br>`theta~ dlnorm(alpha,tau)` | | |
| Gamma<br>$[z\|\alpha,\beta] =$<br>$\frac{\beta^{\alpha}}{\Gamma(\alpha)} z^{\alpha-1} e^{-\beta z}$<br>$\Gamma(\alpha) =$<br>$\int_0^{\infty} t^{\alpha-1} e^{-t}\,\mathrm{d}t\,.$ | The time required for a specified number of events to occur in a Poisson process. Any continuous quantity that is non-negative. | $\alpha =$ shape<br>$\beta =$ rate<br>$\alpha = \frac{\mu^2}{\sigma^2}$<br>$\beta = \frac{\mu}{\sigma^2}$<br>Note–be very careful about rate, defined as above, and scale $= \frac{1}{\beta}$. | $\mu = \frac{\alpha}{\beta}$<br>$\sigma^2 = \frac{\alpha}{\beta^2}$ | `dgamma(x, shape, rate, log = FALSE)`<br>`pgamma(q, shape, rate)`<br>`qgamma(p, shape, rate)`<br>`rgamma(n, shape, rate)` | `#likelihood`<br>`y[i]~ dgamma(r,n)`<br>`#prior`<br>`theta~dgamma(r,n)` | 1) Poisson mean<br>2) normal precision (1/variance)<br>3) $n$ parameter (rate) in the gamma distribution | `dgamma(.001,.001)` |
| Beta<br>$[z\|\alpha,\beta] =$<br>$B\, z^{\alpha-1}(1-z)^{\beta-1}$<br>$B = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}$<br>Because $B$ is a normalizing constant,<br>$[z \mid \alpha,\beta] \propto$<br>$z^{\alpha-1}(1-z)^{\beta-1}$ | Continuous random variables that can take on values between 0 and 1–any random variable that can be expressed as a proportion; survival, proportion of landscape invaded by exotic, probabilities of transition from one state to another. | $\alpha = \frac{(\mu^2-\mu^3-\mu\sigma^2)}{\sigma^2}$<br>$\beta = \frac{\mu-2\mu^2+\mu^3-\sigma^2+\mu\sigma^2}{\sigma^2}.$ | $\mu = \frac{\alpha}{\alpha+\beta}$<br>$\sigma^2 = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$ | `dbeta(x, shape1, shape2, log = FALSE)`<br>`pbeta(q, shape1, shape2, )`<br>`qbeta(p, shape1, shape2, )`<br>`rbeta(n, shape1, shape2)` | `#likelihood`<br>`y[i] ~ dbeta(alpha, beta)`<br>`#prior`<br>`theta ~ dbeta(alpha, beta)` | $p$ in binomial distribution | `dbeta(1,1)` |
| Uniform<br>$[z\|\alpha,\beta] = \begin{cases} \frac{1}{\beta-\alpha} & \text{for } \alpha \le z \le \beta, \\ 0 & \text{for } z < \alpha \text{ or } z > \beta \end{cases}$ | Any real number. | $\alpha =$ lower limit<br>$\beta =$ upper limit<br>$\alpha = \mu - \sigma\sqrt{3}$<br>$\beta = \mu + \sigma\sqrt{3}$ | $\mu = \frac{\alpha+\beta}{2}$<br>$\sigma^2 = \frac{(\beta-\alpha)^2}{12}$ | `dunif(x, min, max,log = FALSE)`<br>`punif(q, min, max)`<br>`qunif(p, min max)`<br>`runif(n, min, max)` | `#prior`<br>`theta~dunif(a,b)` | | $a$ and $b$ such that posterior is “more than entirely” between $a$ and $b$ |