DETERMINISTIC FUNCTIONS CHEAT SHEET

Properties of commonly used functions in ecology

Polynomials	**Form**	**Range**	**Left end**	**Right end**	**Middle**
Line	$y = \sum_{i=0}^{n} a_i x_i$	{-∞, ∞}	$y \rightarrow \pm\infty$; constant slope	$y \rightarrow \pm\infty$; constant slope	Monotonic
Quadratic	$y = \sum_{i=0}^{n} a_i x_i^2$	{-∞, ∞}	$y \rightarrow \pm\infty$; accelerating	$y \rightarrow \pm\infty$; accelerating	Single max/min
Cubic	$y = \sum_{i=0}^{n} a_i x_i^3$	{-∞, ∞}	$y \rightarrow \pm\infty$; accelerating	$y \rightarrow \pm\infty$; accelerating	Up to 2 max/min

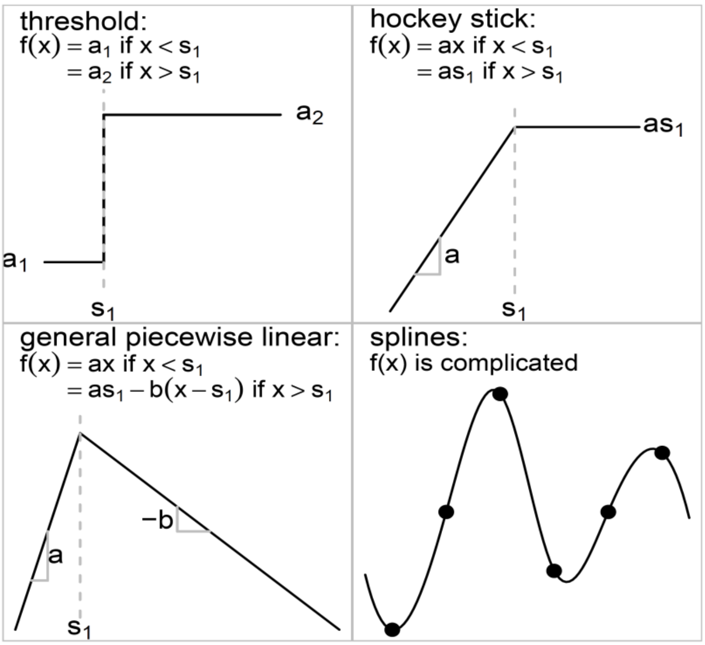
Piecewise polynomials	**Form**	**Range**	**Left end**	**Right end**	**Middle**
Threshold	$y = a_1$ *if* $x<T$ $= a_2$ *if* $x>T$	{-∞, ∞}	flat	flat	breakpoint
Hockey stick	$y = ax$ *if* $x<T$ $= aT$ *if* $x>T$	{-∞, ∞}	flat or linear	flat or linear	breakpoint
Piecewise linear	$y = ax$ *if* $x<T$ $= aT-b(x-T)$ *if* $x>T$	{-∞, ∞}	linear	linear	breakpoint

Rational	**Form**	**Range**	**Left end**	**Right end**	**Middle**
Hyperbolic	$y = \frac{a}{b+x}$	{0, ∞}	$y \rightarrow \infty$ or finite	$y \rightarrow 0$	decreasing
Michaelis-Menten	$y = \frac{ax}{b+x}$	{0, ∞}	$y = 0$, linear	$asymptote$	saturating
Holling Type III	$y = \frac{ax^2}{b^2+x^2}$	{0, ∞}	$y = 0$, accelerating	$asymptote$	sigmoid
Holling type IV (c>0)	$y = \frac{ax^2}{b+cx+x^2}$	{0, ∞}	$y = 0$, accelerating	$asymptote$	hump-shaped

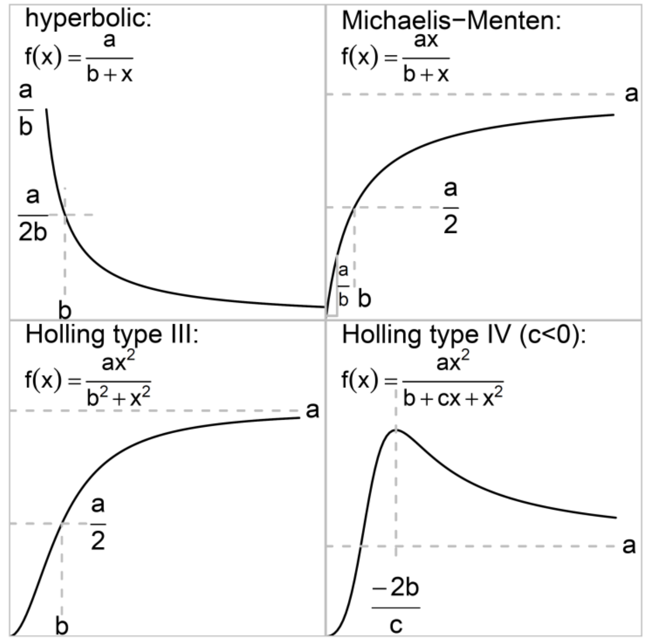
Exponential-based	Form	Range	Left end	Right end	Middle
Neg. exponential	$y = ae^{-bx}$	$\{0, \infty\}$	y finite	$y \to 0$	decreasing
Monomolecular	$y = a(1 - e^{-bx})$	$\{0, \infty\}$	$y = 0$, linear	$y \to 0$	saturating
Ricker	$y = axe^{-bx}$	$\{0, \infty\}$	$y = 0$, linear	$y \to 0$	hump-shaped
Logistic	$y = \frac{e^{a+bx}}{1+e^{a+bx}}$	$\{0, \infty\}$	$y = 0$, accelerating	asymptote	sigmoid

Power-based	Form	Range	Left end	Right end	Middle
Power law	$y = ax^b$	$\{0, \infty\}$	$y \to 0, y \to \infty$	$y \to 0, y \to \infty$	monotonic
Von Bertalanfy	$y = a\left(1 - e^{b-k(a-d)x}\right)^{\left(\frac{1}{1-d}\right)}$	$\{0, \infty\}$	$y = 0$, accelerating	asymptote	sigmoid
Gompertz	$y = e^{-ae^{-bx}}$	$\{0, \infty\}$	$y = 0$, accelerating	asymptote	symmetric
Shepherd	$y = \frac{ax}{b+x^c}$	$\{0, \infty\}$	$y = 0$, linear	$y \to 0$	hump-shaped
Hassell	$y = \frac{ax}{(b+x)^c}$	$\{0, \infty\}$	$y = 0$, linear	$y \to 0$	hump-shaped
Non-rectangular hyperbola	Like MM				

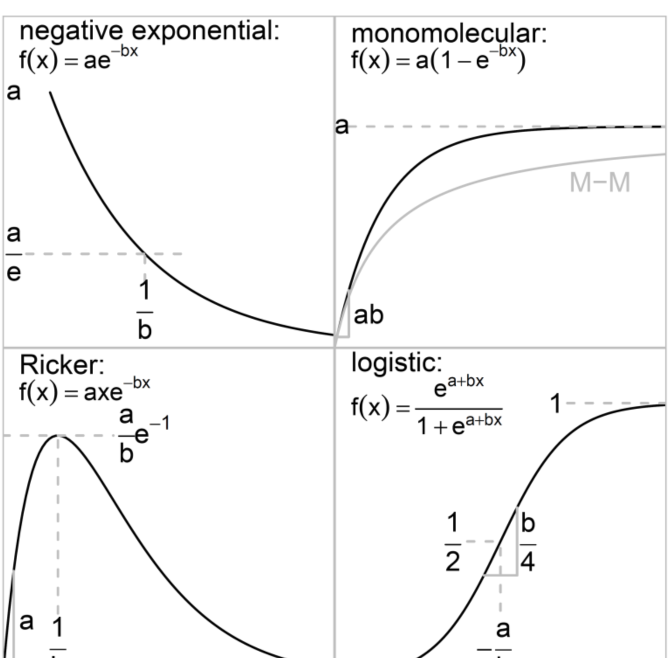
PIECEWISE POLYNOMIALS



RATIONAL:



EXPONENTIAL-BASED



POWER-BASED